# Salomaa's Axioms

Union Rules

1. $r_1 + r_2 =_{\mathcal{L}} r_2 + r_1$
2. $r + \emptyset =_{\mathcal{L}} r$
3. $\emptyset + r =_{\mathcal{L}} r$
4. $r + r =_{\mathcal{L}} r$
5. $r_1 + (r_2 + r_3) =_{\mathcal{L}} (r_1 + r_2) + r_3$

Sequential Composition Rules

1. $r \cdot \varepsilon =_{\mathcal{L}} r$ and $\varepsilon \cdot r =_{\mathcal{L}} r$
2. $r \cdot \emptyset =_{\mathcal{L}} \emptyset$ and $\emptyset \cdot r =_{\mathcal{L}} \emptyset$
3. $r_1 \cdot (r_2 \cdot r_3) =_{\mathcal{L}} (r_1 \cdot r_2) \cdot r_3$
4. $r_1 \cdot (r_2 + r_3) =_{\mathcal{L}} (r_1 \cdot r_2) + (r_1 \cdot r_3)$
5. $(r_1 + r_2) \cdot r_3 =_{\mathcal{L}} (r_1 \cdot r_3) + (r_2 \cdot r_3)$

Basic Kleene Star Rules

1. $\varepsilon + rr^* =_{\mathcal{L}} r^*$
2. $\varepsilon + r^*r =_{\mathcal{L}} r^*$
3. $(\varepsilon + r)^* =_{\mathcal{L}} r^*$

Arden's Rules

- (Left) if $s =_{\mathcal{L}} t + r \cdot s$, then $s =_{\mathcal{L}} r^* \cdot t$
- (Right) if $s =_{\mathcal{L}} t + s \cdot r$, then $s =_{\mathcal{L}} t \cdot r^*$